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THERMAL CONDUCTIVITY OF TRANSPOSED MULTISTRAND FLAT SUPERCONDUCTOR

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UDC 536.21

Longitudinal and transverse thermal conductivities are calculated for a rectangular superconductor consisting of transposed multistrand wires.

Coils for superconducting magnets are often wound with transposed wires, each consisting of twisted superconducting strands embedded in a metal matrix. There are available transposed conductors with the space between strands completely filled by solder metal, in another version of such conductors the strands are only coated with solder metal or an oxide layer.

Inasmuch as such coils are cooled only at certain locations, information about the longitudinal thermal conductivity $\lambda_{\parallel}$ and the transverse thermal conductivity $\lambda_{\perp}$ is needed for an understanding of the processes of heat propagation along the conductor and heat transfer to cooling helium. These thermal conductivities have been calculated on the basis of the conductor model with solder metal between individual multistrand wires (Fig. 1).

Each of the quantities $\lambda_{\perp}$ and $\lambda_{\parallel}$ consists of two components λ_1 and λ_2 characterizing heat conduction through wire and solder, respectively, in directions perpendicular and parallel to the wires. Both λ_1 and λ_2 are determined not only by the thermal conductivity of the respective materials (wire and solder) but also by the coefficients of heat transfer between them and by the geometry of the conductor structure. Furthermore, $\lambda_{\perp}$ and $\lambda_{\parallel}$ depend on the transposition angle of wires in the conductor.

Thermal Conductivity of Composite Wire. In calculation of the thermal conductivity of a composite wire, it is permissible to disregard heat conduction through the superconductor, inasmuch as the thermal conductivity of the matrix material (copper) λ_M is approximately three orders of magnitude higher than the thermal conductivity of the superconducting material (NbTi) λ_C [1]. In effect, therefore, superconductor strands only reduce the total cross section for heat removal.

An expression for the transverse thermal conductivity λ_{M_1} of wire was derived on the basis of the model shown in Fig. 2. The section of a composite wire is subdivided into hexagonal structures with hexahedral superconductor strands at their centers so that the area for heat transfer decreases and the path of heat transfer becomes longer. When a characteristic element of the matrix is subdivided into four geometrical segments, A, B, C, D, then the sum of their thermal resistances will determine λ_{M_1}:

$$\lambda_{M1} = \frac{\pi}{2} A_1 \lambda_M, \quad (1)$$

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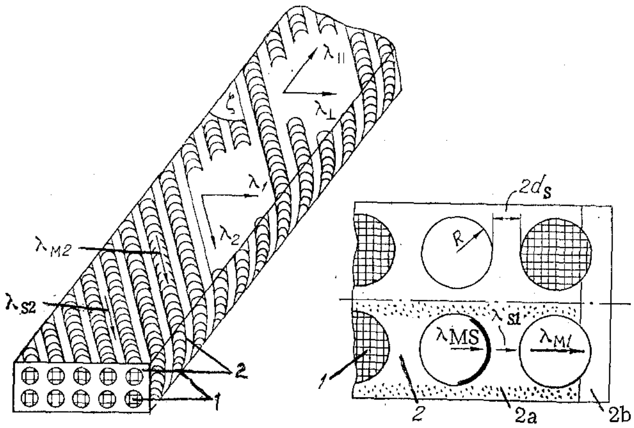

Fig. 1. Schematic representation of transposed superconductor with solder alloy between wires: 1) superconducting multistrand wire; 2) solder; 2a) lateral solder coating; 2b) surface solder layer.

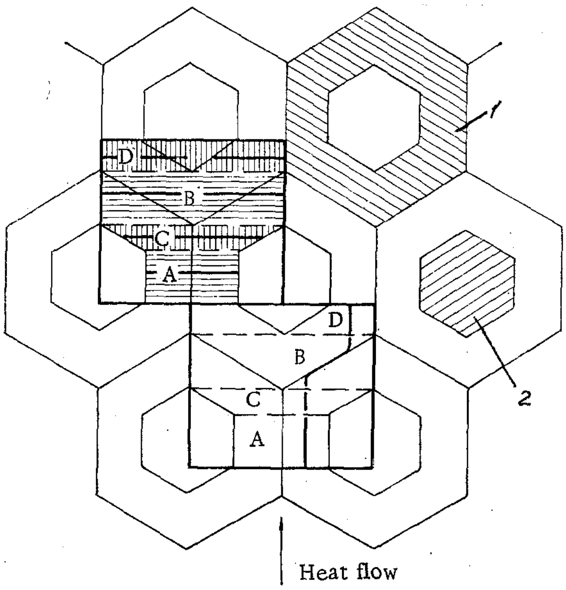

Fig. 2. Model representation of section of superconducting composite wire: 1) metal matrix; 2) superconductor strand; A, B, C, D) segments of characteristic matrix element whose geometrical dimensions determine λ_{M_1} (widths and lengths of structures indicated, respectively, in upper and lower rectangles).

where

$$A_1 \sim \left(\frac{L_A}{F_A} + \frac{L_B}{F_B} + \frac{L_C}{F_C} + \frac{L_D}{F_D}\right)^{-1}$$

is a geometrical factor which depends only on the fraction k_c of wire volume occupied by superconducting material, L_A, L_B, L_C, L_D are the lengths and F_A, F_B, F_C, F_D are the cross-sectional areas for calculation of the thermal resistances of segments A, B, C, D of a characteristic matrix element respectively. The function $A_1 = f(k_c)$ is shown in Fig. 3.

The thermal conductivity of a wire along its axis is

$$\lambda_{M2} = \frac{\pi}{4} A_2 \lambda_M, \tag{2}$$

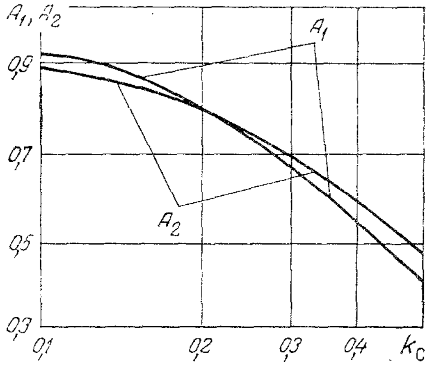

Fig. 3. Dependence of factors A_1 and A_2 on space factor k_c representing fraction of wire volume occupied with superconducting material.

where

$$A_2 = 1 - k_c. \tag{3}$$

The function $A_2 = f(k_c)$ is also shown in Fig. 3. Similar expressions for these quantities are given elsewhere [2].

Thermal Conductivity of Solder. The thermal conductivities of solder λ_{S_1} and λ_{S_2} were calculated corresponding to λ_{M_1} and λ_{M_2} in the respective directions. Its transverse thermal conductivity was calculated according to the method [2] for taking into account such interstices. In an arrangement with the wires in the conductor disposed in parallel rows and with the thermal conductivity of the matrix material one order of magnitude higher than that of the solder, the transverse thermal conductivity of the solder is

$$\lambda_{s1} = \sqrt{\frac{2R}{d_s}} \operatorname{arctg}\left(\sqrt{\frac{2R}{3d_s}}\right)\lambda_s. \tag{4}$$

The thermal conductivity of the solder in the direction parallel to the wires is

$$\lambda_{s2} = \left(1 - \frac{\pi}{4}\right)\lambda_s. \tag{5}$$

Heat Transfer between Wire and Solder. In accordance with the conditions assumed for calculation of λ_{S_1}, the heat transfer between adjoining wire and solder is determined by the contact area and the heat-transfer coefficient α_{MS}

$$\lambda_{MS} = \frac{2}{3}\pi R \alpha_{MS}. \tag{6}$$

Transposition of wires in a conductor makes the thermal path longer than in the preceding case. This involves multiplying λ_{M_1} and λ_{S_1} by cos ζ and λ_{M_2} and λ_{S_2} by sin ζ for calculation of $\lambda_{\perp}$, vice versa for calculation of $\lambda_{\parallel}$. In the calculation of both one can regard λ_{MS} as being independent of the transposition angle ζ of wires in the conductor.

The thermal conductivities $\lambda_{\perp}$ and $\lambda_{\parallel}$ of a transposed multistrand superconductor have been determined on the basis of the following model representations λ_1 and λ_2 characterize heat transfer through the conductor to the outermost wire at the heat emitting surface. Here the thermal fluxes merge and are transmitted farther through the surrounding solder layer of thickness d_S to the very heat emitting surface (Fig. 1).

If m is the number of wires through which the thermal flux successively passes, then the transverse thermal conductivity of the solder is described by the expressions

$$\frac{1}{\lambda_{\perp}(\zeta)} = \frac{1}{\lambda_{1\perp}(\zeta) + \lambda_{2\perp}(\zeta)} + \frac{1}{m}\left(\frac{1}{\lambda_{S1}} + \frac{2}{\lambda_{MS}}\right), \tag{7}$$

$$\frac{1}{\lambda_{1\perp}(\zeta)} = \frac{1}{\lambda_{M1}\cos\zeta} + \frac{2(m-1)}{m}\left(\frac{1}{\lambda_{MS}} + \frac{1}{2\lambda_{S1}\cos\zeta}\right), \tag{8}$$

$$\lambda_{2\perp}(\zeta) = (\lambda_{M2} + \lambda_{S2})\sin\zeta \tag{9}$$

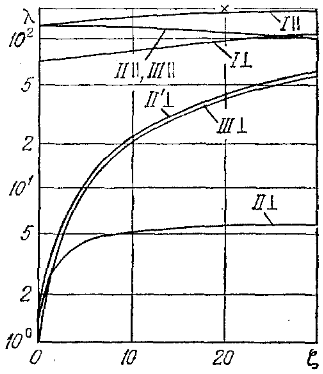

Fig. 4. Dependence of thermal conductivity $\lambda(W \cdot m^{-1} \cdot K^{-1})$ of superconductor on transposition angle ζ (ang. deg) at 4.2°K, for various forms of contact between wires: I) metallic solder; II) epoxy resin; II') epoxy resin but without surface layer of epoxy resion; III) surface-oxidized wires; x) $\lambda_{I\|}$ at T = 5°K, B = 5 T [9].

and its longitudinal thermal conductivity is described by the expressions

$$\lambda_{\|}(\zeta) = \lambda_{1\|}(\zeta) + (\lambda_{M2} + \lambda_{S2}) \cos \zeta, \tag{10}$$

$$\frac{1}{\lambda_{1\|}(\zeta)} = \frac{1}{\lambda_{M1} \sin \zeta} + \frac{2(m-1)}{m} \left(\frac{1}{\lambda_{MS}} + \frac{1}{2\lambda_{S1} \sin \zeta} \right). \tag{11}$$

These expressions are valid for a conductor with rectangular cross section, deviations from such a geometry must be accounted for in the expressions for the individual components of λ_1 and λ_2.

This formalism also facilities the calculation of $\lambda_\perp$ and $\lambda_\|$ for other conductor configurations such as, for example, conductor without solder. The method of determining the individual components λ_{M_1}, λ_{M_2}, λ_{S_1}, λ_{S_2} is, furthermore, suitable for describing the effect of the presence of other materials such as a high-resistivity barrier sheath around the wire or a bimetal wire matrix.

As an example illustrating the effect of solder on the thermal conductivity of a transposed superconductor, we will consider three cases: I) conductor soldered with metallic alloy; II) conductor impreganted with epoxy resin; III) conductor consisting of transposed wires with oxide coatings.

Calculations were based on the expressions given here, with the following general data (T = 4.2°K): $k_C = 0.5$ and thus $A_1 = 0.43$, $A_2 = 0.5$; $\lambda_M = 3 \cdot 10^2$ $W \cdot m^{-1} \cdot K^{-1}$ in magnetic field at 4.5 T [3-6]; 2R - 1 mm, $d_S = 0.01$ mm, m = 6. The data for each specific case were: I) $\lambda_S \sim 10$ $W \cdot m^{-1} \cdot K^{-1}$ for solder on indium base [2], $\alpha_{MS} \sim 10^6$ $W \cdot m^{-2} \cdot K^{-1}$ for soldered junction between copper and indium alloy [7]; II) $\lambda_S \sim 10^{-1}$ $W \cdot m^{-1} \cdot K^{-1}$ for epoxy resin [2], $\alpha_{MS} \sim 10^4$ $W \cdot m^{-2} \cdot K^{-1}$ for junction between copper and epoxy resion [8]; III) contact between oxidized copper surface under pressure less perfect than in cases I) and II) on account of reduced contact area between wires (touching only along limited segments of their perimeters). Calculations were, therefore, based on assuming $\lambda_{MS} \ll \lambda_{M_2}$, i.e., heat transfer through wires only. Heat conduction through the oxide sheath with a thickness of the order of 1 μm could be disregarded, the thermal conductivity of this oxidized conductor being, in the first approximation with an error not exceeding 10%, equal to that of a conductor without solder and oxide.

The results of calculations are shown in Fig. 4, depicting the thermal conductivities of a transposed conductor as functions of the transposition angle ζ. A comparative examination of these curves leads to the following conclusions.

1. The highest thermal conductivity in both direction has a conductor completely soldered with metallic alloy (curves $I\perp$ and $I\|$). The active participation of solder in the heat conduction process is confirmed by the increase of the longitudinal thermal conductivity $\lambda_{I\|}$ with increasing transposition angle ζ, while the longitudinal thermal conductivity of an epooxidized conductor ($II\|$) and of an oxidized conductor ($III\|$).

2. Curve $II\perp$ differs from curve $II'\perp$ describing the transverse thermal conductivity of an epoxidized conductor in that it corresponds to the case of no epoxy surface layer between the outermost wire and helium (Fig. 1 and Eq. (7), second term). Evidently, an outer coating of the conductor with a thermal insulation material reduces the transverse thermal conductivity of an epoxidized conductor appreciably.

3. The thermal conductivities $\lambda_{III\parallel}$ and $\lambda_{III\perp}$ of an oxidized conductor are almost equal to those of an epoxidized conductor $\lambda_{II\parallel}$ and $\lambda_{II'\perp}$ respectively. This can be explained by heat being conducted principally along individual wires in both cases.

The inequality $\lambda_{S_1} \ll \lambda_{M_2}$ holds true in all three cases. Therefore, heat conduction through a thin lateral wire coating (solder or resin, Fig. 1) could be disregarded in calculations of the transverse thermal conductivity.

Very few studies on the thermal conductivity of multistrand wire superconductors have been published in the technical literature. Brechna [2] proposed a method of calculating the temperature distribution in a coil completely impregnated with a thermosetting material. He used $4\cdot10^{-2}$ $W\cdot m^{-1}\cdot K^{-1}$ for the thermal conductivity of this material, pointing out that transposition of wires should make the transverse thermal conductivity of the conductor much higher than that of the impregnating material. In another study Herzog and Malyuk [9] tested a transposed conductor (ζ = 20°) consisting of 12 wires (k_C = 0.42) soldered with an indium alloy, the results yielding $\lambda_{I\parallel} = 1.5\cdot10^2$ $W\cdot m^{-1}\cdot K^{-1}$ at T = 5°K and B = 5 T in close agreement with calculations (Fig. 4).

These examples demonstrate that effective heat conduction in a transposed superconductor, with the usual transposition angle $\zeta \approx 15°$, occurs almost entirely along the individual wires.

The obtained results make it possible to simplify the solution to several problems involved in the study of the characteristics of superconducting pulse magnets. For instance, information about longitudinal and transverse thermal conductivities is needed for determining the propagation of the normal phase or the conductor cooling. An experimental gathering of such data is difficult, especially where the transverse direction is concerned, because of the complexity of a model of the heat conduction process. The formalism presented here makes it possible to determine the thermal conductivities of such a conductor with sufficient accuracy, which has been confirmed by a comparision of experimental and theoretical data. In addition to geometrical characteristics, moreover, calculations require only the thermal conductivities of individual materials to be known and these are much more easy to determine experimentally.

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NOTATION

A_1 and A_2, geometrical factors; d_S, half-thickness of solder between adjacent wires; F, cross section for calculating the thermal resistance; L, length for calculating the thermal resistance; k_C, fraction of wire volume occupied by a superconductor; m, number of wires through which the thermal flux passes successively; R, wire radius; α_{MS}, heat-transfer coefficient at the boundary between wire and solder; ζ, transposition angle of wires in a conductor; $\lambda_\perp$, transverse thermal conductivity and $\lambda_\parallel$, longitudinal thermal conductivity of a conductor; λ_1 and λ_2, thermal conductivities of a conductor respectively along and across the wires; λ_M, thermal conductivity of the wire matrix; λ_C, thermal conductivity of the superconductor; λ_S, thermal conductivity of the solder; λ_{M_1}, transverse thermal conductivity and λ_{M_2} is the longitudinal thermal conductivity of the wires; λ_{S_1}, transverse thermal conductivity and λ_{S_2} is the longitudinal thermal conductivity of the solder in a conductor; and λ_{MS}, thermal conductivity of the junction between wire and solder.

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IDENTIFIABILITY IN THE LARGE OF A LINEAR HEAT-CONDUCTION EQUATION SUBJECT TO CAUCHY BOUNDARY CONDITIONS

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The feasibility of simultaneously determining constant values of the specific heat, thermal conductivity, and heat-transfer coefficient from observations of a unique temperature field is studied.

An important problem in the theory of experimental design is the formulation of conditions such that the number of unknowns can be maximized for a given volume of measurements. In the practice of thermophysical investigations [1, 2] the comprehensive determination of the properties of a body is usually limited to two coefficients: the specific heat and the thermal conductivity. Extending the statement of the problem, we now explore the feasibility of simultaneously determining the values of the specific heat, thermal conductivity, and heat-transfer coefficient from observations of a unique temperature field.

To solve this problem we use an approach proposed earlier [3]. It is based on an investigation of the one-to-one correspondence between the unknown parameters and the state function of the model in question. Then the determination of the class of temperature fields for which the one-to-one correspondence fails could provide the basis for simultaneously identifying several parameters of the thermal model according to the conditions for the elimination of observations of an unidentifiable state. From the point of view of uniqueness of the solution of inverse coefficient problems and within the framework of linear models the present study continues work begun earlier [4], where it was proposed that the conditions for preservation of the one-to-one correspondence be specified as identifiability in the large and the family of coefficients corresponding to one particular solution of the boundary-value problem was expressed as an ambiguity subset.

We now consider a linear heat-conduction equation whose associated boundary conditions contain unknown coefficients. Let it be supposed that the following boundary-value problem is given in the domain of variation of the independent variables $Q_T = \{(x, t) : 0 < x < 1, 0 < t < T\}$:

$$a_1 \frac{\partial u}{\partial t} = a_2 \frac{\partial^2 u}{\partial x^2} + f(x, t), \quad (x, t) \in Q_T,$$

$$u|_{t=0} = \varphi(x), \quad 0 < x < 1,$$

$$a_3 (u|_{x=0} - v_0) - a_2 \left. \frac{\partial u}{\partial x} \right|_{x=0} = 0, \quad 0 < t < T, \qquad (1)$$

$$a_3 (u|_{x=1} - v_1) + a_2 \left. \frac{\partial u}{\partial x} \right|_{x=1} = 0, \quad 0 < t < T,$$

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